The Hadronic Cross–Section in the Resonance Energy Region *

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We study the hadronic vacuum polarization in the resonance energy region, using the framework given by the Resonance Effective Theory of QCD. We consider the incorporation of vector–pseudoscalar meson loops that give, inclusively, three and four pseudoscalar meson cuts. After resummation we achieve a QCD–based inclusive parameterization of the correlator, hence of the hadronic cross–section in the energy region populated by resonances.

1. Introduction

The hadronic spectrum from e^+e^- annihilation in the energy range between 1 and 2 GeV exhibits a rather rich and complex structure. Theoretically, the region $E \gtrsim M_\rho$ (with M_ρ the mass of the $\rho(770)$ resonance), being far away from the chiral domain, is poorly known due to the intricate non-perturbative dynamics of QCD. The conventional approach to extract the hadronic matrix elements of the relevant QCD currents has mainly relied on the available experimental information, such as $e^+e^- \to hadrons$ or semileptonic decays. From these data, the hadronic observables have been obtained either by direct integration of the data or by *ad hoc* parameterizations lousily inspired by QCD. Both approaches have an obvious drawback: they do not tell us much about the physics which lies behind. Even when we can obviate the physical interpretation, we shall keep in mind that fitting (or integrating) procedures inherit all the uncertainties associated to the experimental data, making it very difficult to define the accuracy of the results. An estimation of the theoretical errors introduced by the above techniques is always a matter of discussion. Recall, for example, the running of the QED fine structure constant $\alpha(s)$ and the anomalous magnetic moment of the muon. These are observables whose theoretical predictions are limited by loop effects from hadronic vacuum polarization. Both magnitudes are related via dispersion relations to the hadronic production rate in e^+e^- annihilation, which can be evaluated using e^+e data and hadronic τ decays. It is clear that the apparent discrepancy between the measured value for the anomalous magnetic moment of the muon and the Standard Model prediction requires a careful review of the theoretical uncertainties associated to the hadronic contribution to accurately determine the size of this deviation. An analysis of these observables in a model-independent way could clarify the issue.

Attempts based on effective actions of QCD have achieved a remarkable success in describing the data for energies up to 1 GeV, and indeed suggest that this approach may be continued to higher energies. The pion vector form factor at very low energies has been calculated in chiral perturbation theory, allowing to describe the $e^+e^- \to \pi^+\pi^-$ data in this region very accurately with the known values of the chiral parameters [1]. Concerning the muon anomalous magnetic moment, the use of the chiral expansion for the two-pion contribution at $E \leq 0.5$ GeV has dramatically decreased its error as compared to previous estimations directly obtained from the raw data. At higher energies ($E \sim M_\rho$), the appropriate framework to implement QCD information is Resonance Chiral Theory (RχT). This scheme has been the starting point of several works de-

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voted to the study of the pion form factor in the region close to the $\rho(770)$ mass [2,3,4], which have also implemented features provided by the $1/N_C$ expansion, resummation techniques and other important constraints such as analyticity and unitarity.

Our goal is to provide a QCD–based parameterization of the hadronic cross–section in the resonance driven 1-2 GeV region. To proceed we will derive an expression for the vector–vector current correlator following similar methods to those used in the works just mentioned. With this aim, we outline here the general strategy to follow, mainly focusing in the technical part of the analysis. As we shall see, the practical implementation of our results shall require further investigations. Among other interesting applications, this project could cast some light on the above–mentioned issue of the anomalous magnetic moment of the muon, for which about 90% of the total hadronic contribution comes from the energy region $E \leq 2$ GeV.

A more thorough explanation of the procedure put forward here is given in Refs. [5].

2. The Effective Action of QCD : Resonance Chiral Theory

The low–energy behaviour of QCD for the light quark sector (u, d, s) is known to be ruled by the spontaneous breaking of chiral symmetry that set up the lightest hadron degrees of freedom, identified with the octet of pseudoscalar mesons. The corresponding effective realization of QCD describing the interactions between the Goldstone fields is called Chiral Perturbation Theory [6], and its effective Lagrangian to lowest order in derivatives, $\mathcal{O}(p^2)$, is given by :

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle \ , \tag{1}$$

where

$$\begin{aligned} u_\mu &= i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger] \ , \\ \chi_\pm &= u^\dagger\chi u^\dagger \pm u\chi^\dagger u \quad , \quad \chi = 2B_0(s+ip) \, . \end{aligned} \tag{2}$$

The unitary matrix in flavour space

$$u(\phi) = \exp\left\{ i\frac{\Phi}{\sqrt{2}\,F}\right\} \quad , \tag{3}$$

is a (non-linear) parameterization of the Goldstone octet of fields, identified with the mesons π, K and η. The external hermitian matrix fields r_μ, ℓ_μ, s and p promote the global $\mathrm{SU(3)_R}\times\mathrm{SU(3)_L}$ symmetry of the Lagrangian to a local one, and generate Green functions of quark currents by taking appropriate functional derivatives. The $\mathcal{L}_\chi^{(2)}$ Lagrangian is settled by fixing the unknown F and B_0 parameters from the phenomenology : $F \simeq F_\pi \simeq 92.4\,\mathrm{MeV}$ is the decay constant of the charged pion and $B_0F^2 = -\langle 0|\bar{\psi}\psi|0\rangle_0$ in the chiral limit.

Starting with the $\rho(770)$, the spectroscopy reveals the existence of multiple vector meson resonances participating in the relevant physics up to $E \sim 2\,\mathrm{GeV}$. These can be classified in $U(3)_\mathrm{V}$ nonets and must be included as explicit degrees of freedom in order to describe the hadron dynamics. In this work we will attach to the lightest multiplet of vector resonances participating in the I=1 vector–vector currents correlator. The generalization to several multiplets is rather straightforward [5].

At the lowest order in derivatives, the chiral invariant Lagrangian for the vector mesons and their interaction with Goldstone fields reads [7], in the antisymmetric tensor formulation,

$$\mathcal{L}_\mathrm{V} \, = \mathcal{L}_\mathrm{K}(V) + \mathcal{L}_2(V) \, , \tag{4}$$

with kinetic terms

$$\mathcal{L}_\mathrm{K}(V) = -\frac{1}{2}\langle \nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu} - \frac{M_V^2}{2}V_{\mu\nu}V^{\mu\nu}\rangle, \tag{5}$$

where M_V is the mass of the lowest nonet of vector resonances under $\mathrm{U(3)_V}$, and the covariant derivative

$$\begin{aligned} \nabla_\mu V &= \partial_\mu V + [\Gamma_\mu, V], \\ \Gamma_\mu &= \frac{1}{2}\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger \} \, , \end{aligned} \tag{6}$$

is defined in such a way that $\nabla_\mu V$ also transforms as a nonet under the action of the group. For the interaction Lagrangian $\mathcal{L}_2(V)$ we have

$$\begin{aligned} \mathcal{L}_2(V) &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle \ , \\ f_\pm^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u \, , \end{aligned} \tag{7}$$

with $F_{L,R}^{\mu\nu}$ the field strength tensors of the left and right external sources ℓ_μ and r_μ, and F_V, G_V are real couplings.

The chiral couplings contained in $\mathcal{L}_2(V)$ only concern the even–intrinsic–parity sector. In Ref. [8] it was shown that, up to $\mathcal{O}(p^4)$ in the chiral counting, the effective Lagrangian $\mathcal{L}_{\chi V} \equiv \mathcal{L}_\chi^{(2)} + \mathcal{L}_{\rm V}$ is enough to satisfy the short-distance QCD constraints where vector resonances play a significant role.

Contributions of one–loop two–point diagrams involving a vector and a pseudoscalar mesons provide an inclusive description of exclusive channels with four (4π) or three ($\overline{K}K\pi$) pseudoscalars. The relevant vertices violate intrinsic–parity and are given by the resonance Lagrangian for the odd–intrinsic–parity sector which reads :

$$\mathcal{L}_V^{\rm odd} = \sum_{a=1}^{7} \frac{c_a}{M_V} \mathcal{O}^a_{\rm VJP} + \sum_{a=1}^{4} d_a \mathcal{O}^a_{\rm VVP} \, . \qquad (8)$$

The new operators $\mathcal{O}^i_{\rm VJP}$ and $\mathcal{O}^i_{\rm VVP}$ have been given explicitly in Ref. [9]. The c_a and d_a real couplings, that are not fixed by the underlying symmetry properties are, in principle, unknown. The set defined above is a complete basis for constructing vertices with only one-pseudoscalar; for a larger number of pseudoscalars additional operators may emerge.

In the following we will consider the Effective Action of QCD given by the RχT Lagrangian :

$$\mathcal{L}_{\rm R\chi T} = \mathcal{L}_\chi^{(2)} + \mathcal{L}_{\rm V} + \mathcal{L}_V^{\rm odd} \, . \qquad (9)$$

3. The vector–vector currents correlator : Dyson-Schwinger resummation

The main object of study in this work is the two-point function built from the I=1 part of the electromagnetic current,

$$\begin{aligned} \Pi^{33}_{\mu\nu}(q^2) &= i \int d^4x \, e^{iqx} \langle 0 | \, T[V_\mu^3(x) V_\nu^3(0)] \, |0\rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu}) \, \Pi^{33}(q^2) \, , \end{aligned} \qquad (10)$$

with the vector current given by

$$V_\mu^3 = \frac{\delta S_R^\chi}{\delta v_3^\mu} \, , \qquad (11)$$

being S_R^χ the effective action associated to $\mathcal{L}_{\rm R\chi T}$, and the external vector field $v_\mu \equiv \frac{\lambda^a}{2} v_\mu^a$. Current conservation has been used to extract the tensor structure of the correlator in Eq. (10). The observable quantity we shall derive from the $\Pi^{33}(q^2)$ correlator is the inclusive hadronic cross–section in the I=1 channel:

$$\begin{aligned} R_{had}^{\rm I=1} &= \frac{\sigma^{\rm I=1}(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \\ &= 12\,\pi \, {\rm Im}\, \Pi^{33}(q^2) \, . \end{aligned} \qquad (12)$$

At the one–loop level we consider two different types of absorptive terms that contribute to the imaginary part of the correlator : loops with two pseudoscalars, arising from $\mathcal{L}_2(V)$ and $\mathcal{L}_\chi^{(2)}$, and loops with one internal resonance, given by $\mathcal{L}_R^{\rm odd}$; both can be attached to a vector meson or directly to the V_μ^3 currents to build up the correlator.

We are not interested though in performing the evaluation of the correlator up to one–loop only. The bare resonances acquire a finite width through resummation of quantum loops in perturbation theory. These effects are subleading in the $1/N_C$ counting but must be accounted for to avoid the singularities arising at energies close to the bare pole of resonance propagators. In order to obtain the dynamics in full we would like to resummate all the possible contributions constructed in terms of the one–loop terms explained above. This we do now in turn.

3.1. Two–pseudoscalar meson loops

These contributions have already been taken into account in detail in Refs. [3,4]. The procedure is sketched in Fig. 1 where the vertices, generated by the vector form factor of the pseudoscalar mesons, are given in Fig. 2.

The resummed two-point function finally reads :

$$\begin{aligned} \Pi_{\phi\phi}(q^2) &= \frac{-4\left(1+\frac{F_V G_V}{F^2}\frac{q^2}{M_V^2-q^2}\right)^2 \overline{B}_{22}}{1+\left(1+\frac{2G_V^2}{F^2}\frac{q^2}{M_V^2-q^2}\right)\frac{2q^2}{F^2}\overline{B}_{22}} \\ &\quad + \frac{F_V^2}{M_V^2-q^2} , \end{aligned} \qquad (13)$$

where $\overline{B}_{22} \equiv B_{22}[q^2, m_\pi^2, m_\pi^2] + \frac{1}{2} B_{22}[q^2, m_K^2, m_K^2]$

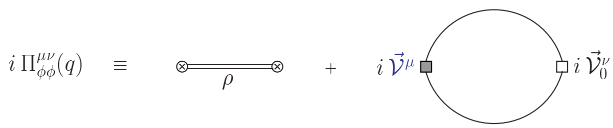

Figure 1. The $\Pi_{\phi\phi}^{\mu\nu}$ vector–vector currents correlator with resummed pseudoscalar loops. Single lines stand for pseudoscalar mesons, double lines stand for vector resonances.

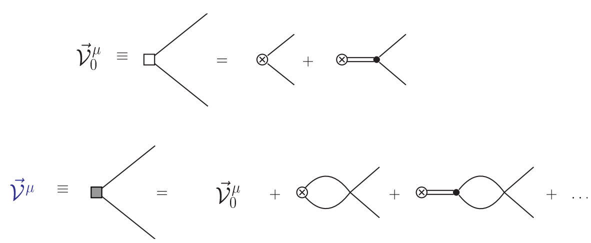

Figure 2. Definition of the off–shell effective current vertices appearing in the resummation $\Pi_{\phi\phi}^{\mu\nu}$ in Fig. 1.

and $B_{22}[q^2, m^2, m^2]$ is the Passarino–Veltman two–point integral as given in Refs. [3,5].

3.2. Vector–pseudoscalar meson loops

These contributions arise from the odd–intrinsic–parity couplings in $\mathcal{L}_V^{\rm odd}$. Here we limit ourselves to the easier case of one multiplet of vector mesons and refer the reader to Ref. [5] for a more complete treatment.

First of all we consider the one–loop contributions. The four allowed topologies are shown in Fig. 3 and the result reads :

$$\Pi_{V\phi}^{1-\ell{\rm oop}}(q^2) = -\sum_{P=\pi,K} \frac{C_P^2}{F^2} \left\{ \frac{F_V^2\, \mathcal{W}_{0,P}(q^2)}{(M_V^2 - q^2)^2} + \mathcal{W}_{1,P}(q^2) + 4\, \frac{F_V\, \mathcal{W}_{2,P}(q^2)}{(M_V^2 - q^2)} \right\}, \tag{14}$$

where the constants C_P are Clebsch–Gordan coefficients depending on the pseudoscalar meson $P = \pi^0, K^+, K^-, K^0, \bar{K}^0$, running inside the loop (together with ω or K^*).

3.2.1. The one–loop functions

The functions $\mathcal{W}_i(q^2)$ are divergent quantities which need to be regularized. The full expression for these functions, obtained following the $\overline{MS}$ subtraction scheme, can be found in Ref. [5], though the renormalization program of RχT remains an unexplored issue.

Alternatively, we can bypass the lack of a consistent renormalization procedure by using a dispersion technique to regularize the real part of the $\mathcal{W}_i(q^2)$ functions from their well–defined imaginary parts :

$$\mathcal{W}_{i,P}(s) = \sum_{k=0}^{N_i} a_i^{(k)}\, s^k + \frac{s^{N_i+1}}{\pi} \int_{s_{th}}^{\infty} ds' \frac{{\rm Im}\, \mathcal{W}_{i,P}(s')}{s'^{(N_i+1)}(s'-s)}\,, \tag{15}$$

where $s_{th} = (M_V + m_P)^2$. The number of subtraction constants needed depends on the behaviour of the spectral densities Im $\mathcal{W}_i(q^2)$ at large q^2 and our evaluation shows that $N_0 = 3$, $N_1 = 1$ and $N_2 = 2$.

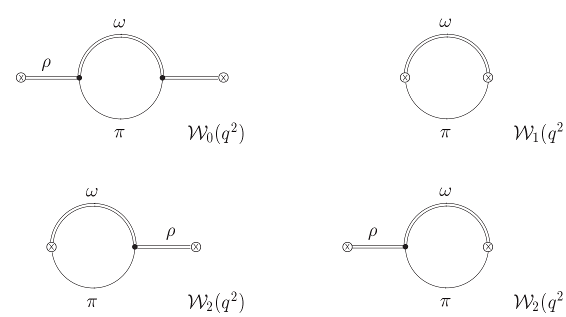

Figure 3. The vector–pseudoscalar mesons contribution to the vector–vector correlator at one–loop. $\mathcal{W}_0$, $\mathcal{W}_1$ and $\mathcal{W}_2$ are the invariant functions associated to the loops according to Eq. (14).

3.2.2. The coupling constants of $\mathcal{L}_V^{\rm odd}$

The one–loop functions $\mathcal{W}_i(q^2)$ depend on the coupling constants c_a and d_a of $\mathcal{L}_V^{\rm odd}$ in

Eq. (8). In Ref. [9] we have put forward a procedure to obtain information on those couplings from QCD itself. This exploits the fact that the QCD Green's function $\langle V_\mu V_\nu P\rangle$ of vector (V_μ) and pseudoscalar (P) QCD currents is an order parameter of the spontaneous chiral symmetry breaking of QCD, hence it does not get perturbative contributions in the chiral limit. Accordingly we match the evaluation of the three–point function in the RχT framework, at leading order in the $1/N_C$ expansion, with the first OPE coefficient of the Green's function within QCD. As a result we fix 5 relations between the unknown c_a and d_a couplings. We have also shown that these results agree well with the phenomenology of odd–intrinsic–parity violating processes.

Quite remarkably the combinations of those couplings appearing in $\mathrm{Im}\,\mathcal{W}_i$ get fixed by the short–distance conditions extracted, using this method, in Ref. [9]. This is an important result of our work because, taking a look at the dispersion relation description in Eq. (15), we realize that the analytic structure of the functions $\mathcal{W}_i$ is completely fixed except for a polynomial whose coefficients are the subtraction constants encoding our lack of knowledge on the renormalization procedure.

3.2.3. Resummation of the one–loop contributions

The inclusion into $\Pi_{\phi\phi}(q^2)$ in Eq. (13) of the resummation driven by the one–loop diagrams in Fig. 3 is rather straightforward if one notices that the role of the latter is to modify structures already given by the two–pseudoscalar meson resummation.

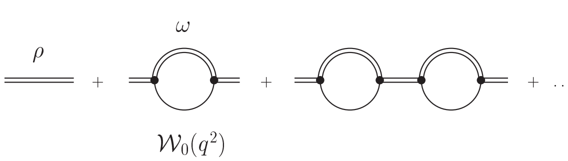

Figure 4. The vector meson propagator with ω-π and K^*-K insertions.

The procedure reduces to consider the following steps :

i) As shown in Fig. 4 the vector–pseudoscalar meson loops modify the propagator of the vector meson. The final effect is to generate a shift of the position of the corresponding pole :

$$M_V^2 \longrightarrow M_V^2 + \mathcal{W}_0(q^2)\,, \tag{16}$$

where

$$\mathcal{W}_0(q^2) = \sum_{P=\pi,K} \frac{C_P^2}{F^2}\,\mathcal{W}_{0,P}(q^2) \tag{17}$$

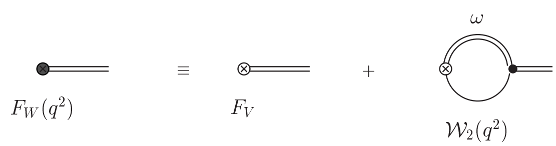

Figure 5. The vertex loop correction to F_V.

ii) The $\mathcal{W}_{2,P}$ functions introduce a q^2 dependence on the coupling of the vector meson to the external current (Fig. 5) :

$$F_V \longrightarrow F_W(q^2) \equiv F_V + \mathcal{W}_2(q^2)\ , \tag{18}$$

with

$$\mathcal{W}_2(q^2) = 2\sqrt{2}\sum_{P=\pi,K}\frac{C_P^2}{F^2}\mathcal{W}_{2,P}(q^2)\,. \tag{19}$$

Including both corrections we finally obtain an analytical expression for the $\Pi^{33}(q^2)$ two–point function which accounts for both the two–pseudoscalar loops and the vector–pseudoscalar meson loops that provide the two–particle absorptive cuts emerged from the odd–intrinsic–parity sector :

$$\Pi^{33}(q^2) = \tag{20}$$

$$\frac{-4\left(1+\dfrac{F_W(q^2)G_V}{F^2}\dfrac{q^2}{M_V^2-q^2+\mathcal{W}_0}\right)^2\overline{B}_{22}}{1+\left(1+\dfrac{2G_V^2}{F^2}\dfrac{q^2}{M_V^2-q^2+\mathcal{W}_0}\right)\dfrac{2q^2}{F^2}\overline{B}_{22}}$$

$$+\frac{F_W^2(q^2)}{M_V^2-q^2+\mathcal{W}_0}+\sum_{P=\pi,K}\frac{C_P^2}{F^2}\,\mathcal{W}_{1,P}(q^2)\,.$$

4. Summary

The expression for $\Pi^{33}(q^2)$ in Eq. (20) is the main result of this study. It provides a QCD–based parameterization of the inclusive hadronic cross–section carrying information on the most relevant exclusive final states of two, three and four pseudoscalar mesons in the isovector channel when only one multiplet of vector mesons is considered.

It is clear that a more complete description of this observable in the whole resonance region ($M_\rho \lesssim E \lesssim 2\,\mathrm{GeV}$) requires the inclusion of heavier multiplets of vector resonances. A look to the RPP [10] shows the existence of three of those multiplets in the $I = 1$ channel commanded by $\rho(770)$, $\rho(1450)$ and $\rho(1700)$. Accordingly, a full description of the vector–vector correlator and the hadronic cross–section needs to consider this circumstance that has been implemented and carried out in detail (for N multiplets) in Ref. [5]. However, in this case, we are not at the point of providing reasonable predictability due to our lack of knowledge on the increasing number of coupling constants that appear as new multiplets are introduced in the RχT.

Nevertheless the procedure deserves a close analysis. When only one multiplet is included, as we have done here, we have managed to obtain information on the couplings from QCD itself (at leading order in the $1/N_C$ expansion) [9] and we have seen that the short–distance conditions fix unambiguously the contributions, but for some polynomials which depend on the regularization procedure in RχT. Consequently it is compulsory to carry the non–trivial study of the implementation of QCD constraints when more than one multiplet are present in order to complement the resummation procedure accomplished here.

Though further investigations are called for, our study has shown that the use of effective theories of QCD in the intermediate energy region, populated by resonances, provides a powerful tool to endow the basic information of the underlying theory into the hadron phenomenology in an essentially model–independent way.

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